91

Reihe Ökonomie

Economics Series

Empirical Performance of the Czech and Hungarian Index Options under Jump

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Abstract

This paper analyses Czech and Hungarian index options that are traded on the Austrian Futures and Options Exchange. We find that the Poisson jump-diffusion and not the GARCH (1,1) process lends statistical support for the data description. We estimate that approximately four-fifth of 4 percent underpricing (for the Czech Index) and 18 percent overpricing (for the Hungarian Index) biases reported for the short term out-of-the-money call options can be explained by the Jump option pricing model. However, we question whether the mispricings from the jump model are operational, especially, in these emerging financial markets.

Keywords

Leptokurtosis, Poisson jump-diffusion, GARCH; equity index options

JEL Classifications

C52, G13, C51, C52

Comments

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Contents

1. Introduction

This paper is motivated by the following two observations. First, the underlying processes for the Czech (CTX) and Hungarian (HTX) index options that are traded on the Austrian Futures and Options Exchange (ÖTOB) exhibit leptokurtosis ("fat tails") in their return distributions.[1] Consequently, it would be interesting to observe operationally significant discrepancies between the pricing models that incorporate fat tails and the ones that are based on Black-Scholes (1973), which critically depends on the log-normal distribution of the terminal stock price. Second, both CTX and HTX provide an interesting platform to analyse the option model that combines both the foreign index and Poisson jump-diffusion features. Both CTX and HTX are different than other foreign index options, say, the Nikkei 225 on the Chicago Merchantile Exchange. That is, the ÖTOB products are measured and actually traded in the U.S. dollars, whereas the Nikkei 225 is measured in Japanese yen but is traded as though it were dollars. There are numerous empirical and theoretical option papers on various versions to Merton's (1976a) option model on Poisson jump-diffusion process.[2] And there are also voluminous work on foreign index derivatives based on Margrabe's (1978) paper.[3] But, to our knowledge, there is no empirical literature that combines

[1] As noted first by Mandelbrot (1963) and Fama (1965), this is one of the undisputed stylized facts in any financial market. Just to name a few, Bollerslev, Chou, and Kroner (1992), Bates (1996b), and Duffie and Pan (1997) all give evidence of leptokurtosis in return distributions for various financial markets around the world.

[2] For example, Ball and Torous (1985), and Jorion (1988) follows the original Merton (1976a) and assume that jump risk is idiosyncratic and thus diversifiable. The others in their models of jump diffusion (Bates, 1991; and Amin, 1993), stochastic volatility jump diffusion (Bates, 1996a, b), and stochastic volatility interest rate jump diffusion (Bakshi, Cao and Chen, 1998) all assume that the jump process is systematic and hence should be priced. Trautmann and Beinert (1995) analyze both cases for the German Stock Exchange (DAX).

[3] For example, Reiner (1992), Dravid, Richardson and Sun (1993), and Wei (1995) solve for the value of various foreign index options given European exercise; Craig, Dravid, and Richardson (1995)

both features.

The purpose of this paper is to develop and empirically investigate a model for pricing the newly created derivative instruments by the ÖTOB utilizing the works by Merton's (1976a) jump-diffusion model and the foreign index option model addressed in Reiner (1992). This paper, theoretically, merely extends Merton's (1976a) one factor jump-diffusion to include two factors. This paper's strength and contribution, however, rest in emprical results and implications. In our empirical approach, we compare our pricing model to a benchmark, namely, Black-Scholes. In doing so, we are not testing the validity of Black-Scholes model for these options as all the strong restrictions implied by the Black-Scholes model are known to be wrong in its details, and formal statistical rejections of the null would tell us no more than we already know. Rather the question we are asking is, how wrong or right is it? In this paper, we investigate the empirical implications of the European CTX and HTX call options that have strike prices in domestic currency (i.e. in U.S. dollars) when jumps are present in the underlying processes.[4] The main reason for focusing on the jump diffusion processes (meaning significant unexpected discontinuous changes in prices) is because skewness and kurtosis specification in returns can easily be decoupled from the volatility.

The Central European Clearing Houses and Exchanges (CECE) under the ÖTOB have created product lines which focus only on East European Index options, and

provide evidence of market efficiency using foreign based derivatives; Dravid, Richardson and Sun (1994) analyze the Yen/Deutsche Mark warrants which are U.S. dollar denominated.

[4]Heavy tails could also be captured by using other approaches such as GARCH, a mixture of distributions and Levy distribution.

began trading these products since March 1997.[5] In this paper, we analyze only the Czech (CTX) and Hungarian (HTX) index options since they provide the longest time series and, so far, the two most liquid products. And further, these two markets provide two distinct and different characteristics: one is bearish (CTX) and the other is bullish (HTX).

Our results can be summarized as follows. We find that, when making pair-wise and combined comparisons between the pure diffusion process and one which includes either a Poisson jump process or a time varying variance, the statistical evidence lend support for the Poisson jump process to describe the data. Consequently, taking the Poisson jump as the underlying process for our option pricing model, we find that the differences in option pricing for the Czech and Hungarian index options arise when the jump model is compared to the model without the jumps. Using the estimated parameters, approximately four-fifth of 2.4 percent (for CTX) and 3.4 (for HTX) percent underpricing biases reported for the short term at-of-the-money call options can be explained by the Jump option pricing model. However, we note that these pricing errors are quite small. Consequently, we question whether these mispricings can be operational when the underlying markets for the traded derivatives are illiquid and have transaction costs in the range of 3-6 percent per trade.

We begin our analysis in the next section with some facts regarding the Czech and Hungarian Traded Indices, and then we provide some statistical evidence that

[5] The CECE product line includes options on the Hungarian Traded Index (HTX) and Czech Traded Index (CTX), the Polish Traded Index (PTX), the Slovakian Traded Index (STX), the Russian Traded Index (RTX) and the CECE Index. The HTX, CTX, PTX, and RTX have been trading since March, May, July 21 ,and December 11, 1997 respectively.

motivate us to investigate models which emphasize the role of Poisson jumps in determining the fatness of the return distributions. In section 3, we present the environment for the underlying processes and describe the method to estimate the parameters for various models. Empirical results for section 3 are given in section 4. In section 5, we outline various payoffs and present empirical pricing implications from the models. We first start with the benchmark "standard" Black-Scholes model on equity index which addresses the case where the product of the foreign asset price and the exchange rate at expiry that is important to the investor.[6] We then analyze the modified Black-Scholes model that includes the Poisson jump-diffusion process. Concluding remarks are presented in section 6.

2. Some Facts about CTX and HTX

We begin with a brief description of CTX and HTX. We then present some descriptive statistical facts about CTX and HTX that motivated our academic interest.

2.1. Product Description of CTX and HTX

The Austrian Futures and Options Exchange has introduced and been trading HTX since March 1997 and CTX since May 1997. The aim of the ÖTOB in creating these derivative products is to provide "efficient risk management tools for the Central and

[6] There are other combinations of options with differing degrees of protection against adverse moves in exchange rates and equity prices. For example, a Quanto Option allows an investor to capture all the upside returns from his/her foreign investment, but removes all the foreign exchange risk by fixing now (at t) the exchange rate that will be applicable to the final payoff at T. Further, an Equity-linked foreign exchange option gives an investor full unhedged exposure to foreign equities but limits the downside of foreign exchange risk. Each of these four options can be valued in closed-form under the Black-Scholes assumptions. The reason being that the payoffs are products of variables that are log-normally distributed; therefore the payoffs are also log-normally distributed.

Eastern European stock markets and make them available to international investors and broker/dealers in the near future" (ÖTOB, 1996). The full description of CTX, HTX, and other CECE product line is clearly detailed in ÖTOB (1996).[7]

For comparison exposition, the following contract description brings out the differences between the CME Nikkei 225 contract and CECE products such as CTX and HTX

	CME Nikkei Contract	ÖTOB CECE
Contract size	USD5×index	index (in USD)
Value if 1 points	USD5	USD5
Tick size	5	5
Tick value	USD25	USD25

And the payoffs of the above two contracts can be written as

CME Nikkei: $(S^{*} - K) \cdot 5USD$

ÖTOB CECE $\left(S^{'*} - K^{'}\right)$

where S^{*} is the terminal value of Nikkei 225 index in JPY, $S^{'*}$ is the terminal value of CECE index in USD, K is the strike price in JPY, and $K^{'}$ is the strike price

[7] We outline some of the major aspects of CTX and HTX in the following.

- The indices are value-weighted in the U.S. dollars.
- The indices are based only on the Blue-Chip stocks, and hence they reflect the current market situation.
- The stock selection criteria are size, liquidity, and representativeness (balance sector structure).
- The Indices generally cover over 60% of the respective market capitalization.
- The Indices are highly correlated with already established broader indices. For example, the correlations between CTX and the Prague Exchange 50 (PX50) and between HTX and the Budapest Exchange (BUX) are 0.784 and 0.787 respectively.

in USD.

CTX and HTX are the main underlying time series to be explained for our options price analysis,

see Figure 1 in appendix.

Figure 1 graphs daily time series of CTX and HTX from January 1 1995 till June 20 1997. For the first seven months, CTX took a major downward course, losing approximately 300 points. Afterwards, there were a few small up and downward swings till the beginning of 1996 when CTX took a momentum and showed a large upward course. From the middle of 1996 till February 1997, there were major boom and bust cycles with the last major peak occurring in February 1997. It then takes a drastic downward course and bottoms out at the beginning of June 1997. These large downward swings are also noted by some of the market commentators for the Czech markets, who often refer to the lack of transparency in the second wave (large scale firm) privatization as a major factor undermining investors' confidence in the equity market, highlighted by a number of high profile controversies over insider trading. One of the major factors for the downward swing in the Czech market is actually due to the privatization program itself: The majority of the firms are privatized involuntarily through political and legal channels. Consequently, although, these firms are now "privatized", most of them are not supported by their management, and thus maintain a firm structure that lacks the incentive schemes.

On the other hand, the HTX displays a different picture than the CTX. Most of the firms in Hungary, on the contrary to the Czech case, are privatized through

Initial Public Offerings. Consequently, these firms' objectives are based on the micro-level: Profitability. As a result, there is a clear upward trend in the index. Until January 1996, HTX only showed small up and downward cycles with the averaged index points around 600. The bullish trend with a few small movements can be observed from the beginning of 1996. A major upturn occurred at the end of 1996 with a sharp downward course bottoming out with 1450 index points in March 1997. HTX took a strong rebound afterwards and climbed to a new record at the end of June 1997. A negative trend does not seem immediate for HTX.[8]

2.2. Descriptive Statistical Facts about CTX and HTX

Table 1 provides descriptive sample statistics for returns on the Czech indices CTX, CTL and CZK and the Hungarian indices HTX, HTL and HUF. CTX and HTX are the returns on the indices based on the $US, while CTL and HTL are based on the local currency. CZK is the return on the exchange rate between the Czech Crown and the $US and finally HUF is the return on the exchange rate between the Hungarian Forint and the $US. To establish a statistical confirmation of the skewed and leptokurtic form of the distributions, we also report test statistics for skewness and kurtosis in Table 1,

see Table 1 in appendix.

As expected the test statistics further lends support to non-normality distribution. The p-values in Table 1 clearly indicate that the null hypothesis of zero skewness

[8] As of February 26, 1998, the closing index value for the HTX is 2152; whereas for the Czech, it is at 618.

and kurtosis cannot be supported. In fact the test statistics for excess kurtosis of returns give a strong support of leptokurtosis and hence fat tails. This support is not surprising for the Czech and Hungarian markets as other equity and currency markets also display fatter than normal tails with mostly negative skewness. (Duffie and Pan, 1997). Moreover, the values in Table 2 confirm certain stylized facts about the statistical properties of financial returns: squared returns are often significant at the first order-correlation (suggesting, the non-linearity in the errors terms) and returns have small autocorrelation, implying that there is almost no predictability in returns over a high frequency sample,

see Table 1 in appendix.

3. Underlying Processes

In this section, we outline the environment for our model, and then present various processes that we use to determine the fit of our data. We also include GARCH(1,1) process as an alternative approach to capture heavy tails. Lastly, we provide the estimation procedures for the processes.

3.1. Environment and Notations

All processes considered in this paper are defined on a common filtered probability space $(\Omega, \Im, P)$, where the filtration $\Im$ is assumed to be the P augmentation of the natural filtration generated by a 2-dimensional Brownian motion $W = \left(W_t^1, W_t^2\right)$. Let there be four securities, two risky and two riskless. The first risky security, S_t, does not pay dividends. Let the price of index S_t, expressed in, say, Hungarian Forint, follow a geometric Brownian motion:

$$\frac{dS_t}{S_t} = \mu_S dt + \sigma_S dW_t^1 \tag{3.1}$$

where μ_S and σ_S are constant real numbers. The σ_S is the instantaneous standard deviation, and μ_S is the instantaneous expected value of the rate of return of the risky asset.

Let the other risky asset, say, exchange rate E_t with the U.S. dollar (i.e. E_t is the value of unit Forint in the \$U.S. at time t) also follows a geometric Brownian motion:

$$\frac{dE_t}{E_t} = \mu_E dt + \sigma_E dW_t^2 \tag{3.2}$$

where, σ_S and σ_E are also constant real numbers. Suppose further that there are no transaction costs and no free lunches in this economy. But, shortselling the security and borrowing or lending in dollars at a constant rate r_d and at another constant rate r_f are permitted. These constant rates, r_d and r_f are the discount rates for the riskless assets, which pay no dividends and they have price processes

$$B_t^d = \exp\{r_d t\}\,,\ t \geq 0 \qquad B_t^f = \exp\{r_f t\}\,,\ t \geq 0 \tag{3.3}$$

Then, we express the bond price process as

$$\frac{dB_t^d}{B_t^d} = r_d\left(S_t, t\right) dt, \qquad \frac{dB_t^f}{B_t^f} = r_f\left(S_t, t\right) dt \qquad 0 \leq t \leq T \tag{3.4}$$

Since both the CTX and HTX are valued and sold in the U.S. dollars, the investor looks at the diffusion process of a product, $V_t = E_t S_t$. Using Ito's lemma on V_t, the diffusion process for V_t is then

$$\frac{dV_t}{V_t} = \mu_V dt + \sigma_V dW_t^V \tag{3.5}$$

where $\mu_V = (\mu_S + \mu_E + \rho_{SE}\sigma_E\sigma_S)$, ρ_{SE} is the instantaneous correlation between the two Wiener processes dW_t^2 and dW_t^1, $\sigma_V = \left(\sigma_E^2 + \sigma_S^2 + 2\rho_{SE}\sigma_E\sigma_S\right)$, and $dW_t^V = \left(\frac{\sigma_E dW_t^2 + \sigma_S dW_t^1}{\sigma_V}\right)$. And using, once again, the Ito's lemma to $\log V_t$

$$d\log V_t = \left(\mu_S + \mu_E - \frac{1}{2}\left(\sigma_E^2 + \sigma_S^2\right)\right) dt + \sigma_V dW_t^V \tag{3.6}$$

In other words, V_t is distributed as log-normal with the mean of $\left(\mu_S + \mu_E - \frac{1}{2}\left(\sigma_E^2 + \sigma_S^2\right)\right)$ and the variance of $\left(\sigma_E^2 + \sigma_S^2 + 2\rho_{SE}\sigma_E\sigma_S\right)$.

3.2. Poisson Jump Diffusion Model

The Poisson jump-diffusion model of Merton (1976a) has two appealing attributes in dealing with financial markets, especially for thinly traded markets. First, this process can explain the observed empirical characteristics of stock and currency returns distributions such as "fat tails" and skewness as was seen in section 2.2. Second, it allows the returns to have certain unexpected "jumps". This fact is also economically appealing since the resulting sample path for the stock price S or exchange rate E will be continuous most of the time, with finite jumps of differing signs and amplitudes occurring at discrete points in time (e.g. stock market crash or unexpected currency devaluation).

The simplest form of jump diffusion model is when the logarithm of the size of the proportional jump has a normal distribution. For the jump-diffusion model, one simply adds a Poisson jump component to the diffusion model. Thus, one replaces geometric Brownian equation such as (3.1) with

$$\frac{dX_t}{X_t} = (\mu_X - \lambda_X \kappa_X)\, dt + \sigma_X dW_t^X + dq_t^X \tag{3.7}$$

where $X = \{S, E\}$, μ_X is the expected return from equity, σ_X is the instantaneous variance of the return conditional on the Poisson event not occurring, and q is a Poisson process generating the jumps. dq^X and dW^X are assumed independent. λ_X is the mean number of jumps per unit time, $\kappa_X \equiv \epsilon\left(Y^X - 1\right)$ is the average jump size (as a % of equity price) where $Y^X - 1$ is the random variable percentage change in the equity price if the Poisson event occurs and ϵ is the expectation operator over the random variable Y^X. And let the $Ln(Y^X)$ be normally distributed with mean θ_X and variance δ_X^2. Clearly, dq^X part describes the part due to the "abnormal" price vibrations. And if $\lambda_X = 0$ (and thus $dq^X = 0$) then the return dynamics would be identical to either equation (3.1) or (3.2).

Using Ito's lemma for the continuous part and an analogous lemma for the jump part[9], a counterpart to equation (3.5), when both the index and exchange rate evolve according to equation (3.7), is as follows

[9] Here, we assumed that the process $V = SE$ is independent of Y. Further, dq_t^S and dq_t^E are independent to each other, and to the Browian motions dW_t^S and dW_t^E. For a description of the corresponding lemma for Poisson processes, see Merton (1971) or the original version in Kushner (1967, page 18-22). And for applications and examples see Shimko (1992).

$$\frac{dV_t}{V_t} = \left(\mu_V - \lambda_V \kappa_V + \frac{\lambda_S \epsilon \left[V_t Y^S\right] + \lambda_E \epsilon \left[V_t Y^E\right]}{V_t}\right) dt + \sigma_V dW_t^V \quad (3.8a)$$

$$= (\mu_V - \lambda_V \kappa_V)\, dt + \sigma_V dW_t^V + dq_t^V \quad (3.8b)$$

where $\lambda_V \kappa_V = \lambda_S \kappa_S + \lambda_E \kappa_E$, $dq_t^V = \left(dq_t^S + dq_t^E\right)$,

Then, V has a log-normal variance of $v_n^2 \equiv \widetilde{\sigma_{SE}^2} + \frac{n\left(\delta_S^2 + \delta_E^2\right)}{T-t}$, where v_n^2 is the total variance of the Poisson jump-diffusion process, and $\widetilde{\sigma_{SE}^2}$ is the volatility coming only from the diffusion part of SE.

The parameters of interest are estimated by numerical maximization of the likelihood function of the parameter vector γ given the observations x, $L\,(X|\gamma)$. Given a sample of daily returns, $X \equiv (x_{1,} x_2, \cdots, x_n)$, the truncated[10] logarithm of the likelihood function for the jump-diffusion process described above is

$$\ln L\,(X|\gamma) \equiv l_j = -T\lambda - \frac{T}{2}\ln(2\pi) + \sum_{t=1}^{T} \ln\left[\sum_{n=0}^{10} \frac{\lambda^n}{n!} \frac{1}{\sqrt{\sigma^2 + \delta^2 n}} \exp\left(\frac{-\left(x_t - \mu - \theta n\right)^2}{2\left(\sigma^2 + \delta^2 n\right)}\right)\right] \quad (3.9)$$

where $\gamma \equiv \left(\lambda, \theta, \delta^2, \mu, \sigma^2\right)$.

3.3. GARCH(1,1) Model

Another well known major source of fat tails is time-varying volatility. Since the introduction of the Autoregressive Conditional Heteroscedasticity (ARCH) of Engle (1982),

[10] As in Ball and Torous (1985), we use N=10. We have also found through various experimentation that N=10 provides satisfactory accuracy for all parameter values that are listed in this section.

there have been many extensions including GARCH (Bollerslev, 1986).[11] Many modelers and practitioners have turned to GARCH(1,1) as a benchmark process in the class of ARCH models (e.g. Duffie and Pan, 1997; J.P.Morgan RiskMetrics, 1996). If x_t represents the actual return at time t and μ is the average return then the return generating process for the GARCH (1,1) volatility model assumes that

$$\begin{aligned} x_t &= \mu + \varepsilon_t \qquad \varepsilon_t \sigma_t^{-1} \sim \mathcal{N}(0,1) \\ \varphi_t &\equiv E_{t-1}\left(\sigma_t^2\right) = \alpha + \beta \left(x_{t-1} - \mu\right)^2 + \eta \sigma_{t-1}^2 \end{aligned} \tag{3.10}$$

where α, β, and η are positive constants with the "non-explosive" condition that $\phi \equiv \beta + \eta < 1$. The variance today depends upon three factors: a constant, yesterday's forecast variance (the GARCH term), and yesterday's news about volatility which is taken to be the squared residual from yesterday (the ARCH term). A high η or β implies a high carry-over effect of past to future volatility (i.e. η or β measures the persistence in volatility, or capture volatility clustering: large returns are more likely to be followed by large returns of either sign than by small returns), while a low η or β implies a heavily damped dependence on past volatility.

The parameters for GARCH (1,1) are also estimated by numerical maximization of the likelihood function. The logarithm of the likelihood function for the GARCH

[11] Other extension, just to name a few, are Integrated GARCH (IGARCH), Exponential GARCH (EGARCH), Cross-Market GARCH, and Regime Switching ARCH (SWACH). (see, Bollerslev, Chou, and Kroner, (1992) for a review of the ARCH literature.

process described above is

$$\ln L\left(X|\overbrace{\gamma}\right) \equiv l_g = -\frac{T}{2}\ln(2\pi) + \sum_{t=1}^{T}\ln\left[\frac{1}{\sqrt{\varphi_t}}\exp\left(\frac{-\left(x_t-\mu\right)^2}{2\varphi_t}\right)\right] \tag{3.11}$$

where $\overbrace{\gamma} \equiv \{\alpha, \beta, \eta\}$.

3.4. Jump and GARCH (1,1)

Since both the Jump diffusion and GARCH(1,1) processes could give arise to having fatter tails than those expected from a normal distribution, we combine both processes to analyze which of the two processes provides a superior description of the data. Consequently, when returns follow both processes, it can be written as follows

$$\frac{dX_t}{X_t} = \left(\mu_X - \lambda_X \kappa_X\right) dt + \varphi_t^X dW_t^X + dq_t^X \tag{3.12}$$

Once again, the parameters of interest are estimated by numerical maximization of the likelihood function of the parameter vector γ given the observations x, $L(X|\gamma)$. Given a sample of daily returns, $X \equiv (x_1, x_2, \cdots, x_n)$, the truncated logarithm of the likelihood function for the jump-diffusion with GARCH (1,1) processes described above is

$$\begin{aligned} \ln L\left(X|\widetilde{\gamma}\right) \equiv \; & l_j = -T\lambda - \frac{T}{2}\ln(2\pi) \\ & + \sum_{t=1}^{T}\ln\left[\sum_{n=0}^{10}\frac{\lambda^n}{n!}\frac{1}{\sqrt{\varphi_t+\delta^2 n}}\exp\left(\frac{-\left(x_t-\mu-\theta n\right)^2}{2\left(\varphi_t+\delta^2 n\right)}\right)\right] \end{aligned} \tag{3.13}$$

where $\widetilde{\gamma} \equiv \left(\alpha, \beta, \eta, \lambda, \theta, \delta^2, \mu, \sigma^2\right)$

3.5. Likelihood Ratio Tests

Maximum-Likelihood Estimation (MLE) not only gives the consistent and invariant estimates, but also permits formal tests of the relative fit of various distributions. For example, since a pure diffusion model is nested within the Poisson jump-diffusion model, a likelihood ratio test can be used to test the null hypothesis H_0: returns are normally distributed. The likelihood ratio statistic is given by

$$\Lambda = -2\left[\ln L\left(X|\gamma\right) - \ln L\left(X|\widehat{\gamma}\right)\right] \tag{3.14}$$

where, $L\left(X|\widehat{\gamma}\right)$ is the likelihood function for the normal distribution of a pure diffusion process, where the parameter vector $\widehat{\gamma} = \left(\mu, \sigma^2\right)$.[12]

4. Empirical Results

For our empirical analysis, we used the following data sources. All the data are daily and start from January 4, 1995 till July 1, 1997. These were obtained from the Austrian Futures and Options Exchange. The U.S. 3-months T-Bill rate is used as a proxy for the risk-free rate and easily can be obtained from the Citibase (FYGN3)[13]. The return series for these indices are calculated by using the differences of price in

[12] The logarithm of the likelihood function $L\left(X|\widehat{\gamma}\right)$ for the normal distribution with the parameter vector $\widehat{\gamma} = \left(\mu, \sigma^2\right)$ can be written as $\ln L\left(X|\widehat{\gamma}\right) \equiv l_N = -\frac{T}{2}\ln(2\pi) + \sum_{t=1}^{T}\ln\left[\frac{1}{\sqrt{\sigma^2}}\exp\left(\frac{-(x_t-\mu)^2}{2\sigma^2}\right)\right]$

[13] The 3-months T-Bill is monthly, which is the average of daily figures. The Board of Governors of the Federal Reserve System is the main source for this data series.

logs, $\ln(p_t/p_{t-1})$.

We begin our empirical analysis with the maximum likelihood estimations for various models and their statistical tests. Also in this section, we compare different models for the fit of the data.

4.1. Maximum Likelihood Estimations and Ratio Tests

Table 3 and Table 4 show the estimated coefficients as well as the empirical fit of the first three models using the CTX and HTX daily return series,

see Tables 3 and 4 in appendix.

For the jump model, as in Ball and Torous (1985), we used the Bernoulli jump-diffusion maximum likelihood estimates as a starting values for the Newton-Raphson algorithm. Consequently, for our initial values, we set $\theta = 0$ and take other parameters arbitrarily. For the GARCH(1,1) model, we used the ordinary least square estimates as the initial values in our maximum likelihood estimation.

Analysis of Table 3 provides an interesting results. For the jump model, the estimated $\widehat{\lambda}$ tells us that there are approximately 0.65 jumps per day for both CTX and HTX. This implies that there is a jump on average every 1.5 days. These estimated $\widehat{\lambda}$s are statistically significant. Although these jumps are statistically significant, one should note that not all of these $\widehat{\lambda}$s represent dramatic jumps or crashes of the markets. If one looks at the implied jump sizes, they are relatively quite small. This result indicates that news typically comes in "packets" which, while small, still have what Merton (1976) calls "non-marginal" effects on underlying securities prices. These frequent but small non-marginal innovations should be distinguished from the infrequent but have "stock market crash" type shocks which, due to the very short time horizon analyzed here, were not observed in our sample space.

For CTX, the average daily return that is due to jump, $\widehat{\lambda\theta}$, is -0.00031. This accounts for approximately 116% of the total average daily return[14]. The estimate $\widehat{\lambda\theta}$ for HTX is 0.000674, which accounts for approximately 90% of the total average daily return. On the volatility estimates, since the total unconditional volatility of the Poisson jump process (annualized with 250 trading days) is given by, $\sigma_{Total} \equiv \sqrt{\left[\widehat{\sigma}^2 + \widehat{\lambda}\left(\widehat{\theta}^2 + \widehat{\delta}^2\right)\right] \cdot 250}$, we can also recover the total variance which is due to the jump component, namely, $\sigma^2_{Jump} \equiv \frac{\widehat{\lambda}\left(\widehat{\theta}^2 + \widehat{\delta}^2\right)}{\left[\widehat{\sigma}^2 + \widehat{\lambda}\left(\widehat{\theta}^2 + \widehat{\delta}^2\right)\right]}$. Thus, for CTX, approximately 78% of the total volatility comes from the jump component σ^2_{Jump}. And for HTX, approximately 80% of the total volatility is accounted by the jump component.

Turning now to the GARCH (1,1) estimation, we can statistically conclude that for both CTX and HTX, the estimates are in line with the previous studies in the literature: GARCH (1,1) provide significant in-sample parameter estimates and highly persistent. All three factors, a constant, yesterday's forecast variance and yesterday's news about volatility, of the GARCH model are statistically significant with $\widehat{\eta}+\widehat{\beta} \approx 1$. One of the interesting notes is the fact that the magnitude of the estimates on $\widehat{\beta}$ and $\widehat{\eta}$ is inversed for CTX and HTX. The persistence in volatility, $\widehat{\eta}$, for CTX (0.77) is much higher than for HTX (0.05). However, the estimate for the yesterday's forecast

[14] The reason for the value which is more than 100% is because the CTX has a downward trend in its returns. In other words, the jump component return is negative and larger than the return for the diffusion part.

variance, $\widehat{\beta}$, for CTX (0.13) is much lower than for HTX (0.38).

Because both the jump and GARCH (1,1) estimates are statistically significant, we also estimate the combined model of jump diffusion and GARCH (1,1) for both the CTX and HTX. The results are given in the fifth and ninth columns of Table 3. As expected, since both the jump and time varying volatility are two major sources of fat tails in return distributions, all the components from the both processes are identified and provide reasonable results. More interesting question is which of the two processes provide a better fit of the data. We address this question using the Likelihood Ratio tests in section 3.5.

The empirical fit of the various stochastic processes for all the returns are presented in Table 4. The hypothesis of a pure diffusion process is rejected against the jump diffusion, GARCH (1,1), and the combined models: The marginal significance level of the χ^2_{d-2}'s are all zero for the both CTX and HTX. However, since neither the jump diffusion nor GARCH(1,1) processes have been explicitly identified in daily return movements, we have performed a nested likelihood test between the combined model versus the jump diffusion and GARCH. In doing so, we would further have a better understanding of the true underlying process. Even with the likelihood ratio tests using the combined likelihood statistics, we cannot statistically conclude that either the jump diffusion or GARCH (1,1) describes the data: The marginal significance level of the χ^2_{d-2}'s are all zero for the both CTX and HTX. However, we note that the values of χ^2_{d-2}'s for the combined versus jump is much lower (63.5 for the CTX and 81.2 for the HTX) than the values of χ^2_{d-2}'s for the combined versus GARCH (1,1) (111.4 for the CTX and 117.9 for HTX).

Since we do not have a nested likehood statistics that would permit us to test directly the null of Jump versus GARCH,we present further statistical results to lend support against the GARCH (1,1) by estimating the ex-post square return - volatility linear regression of the form

$$(x_t - \mu)^2 = \beta_0 + \beta_1 \widehat{\sigma}_t^2 + \eta_t \tag{4.1}$$

where x_t are the observed returns, $\widehat{\sigma}_t^2$ is the predicted variance using the estimated parameters from the GARCH (1,1), and η_t is a white noise. There are numerous studies that show a high degree of volatility persistence and significant in-sample parameter estimates resulting from GARCH (1,1). (e.g. Bollerslev, Chou and Kroner, 1992; Bollerslev, Engle and Nelson, 1994; Shephard, 1996). If GARCH (1,1) is the true process then when the square return regression is run, R^2 should be relatively high, and both $\widehat{\beta_0}$ and $\widehat{\beta_1}$ should be statistically significant and equal to zero and unity respectively. However, there are also number of studies which shows that standard volatility models (i.e. ARCH and GARCH) explain little of the variability in ex-post squared returns in various forms of equation (4.1). (e.g. Cumby, Figlewski and Hasbrouck, 1993; Figlewski, 1997; and Jorion, 1995, 1996). Most of these papers present R^2's from the square return - volatility regressions in the range of 1 - 10 %, and the estimated parameters $\widehat{\beta_0}$ and $\widehat{\beta_1}$ are imprecise. Consequently, these low R^2's in the literature have lend support that the standard GARCH models may be mis-specified and provide limited use.[15]

[15] Andersen and Bollerslev (1997) show that under a different construction of the ex-post squared return - volatility regression using a continuous time framework and high-frequency intradaily data,

Table 5 presents the regression results,

see Table 5 in appendix.

We conclude that using daily returns, we cannot support GARCH (1,1) as the model describing the underlying processes. All the reported R^2's are in the range of 1 - 10%, which are in line with previous studies in the literature, and the estimated $\widehat{\beta_0}$ and $\widehat{\beta_1}$ do not support the hypothesis that they are equal to zero and one respectively. Perhaps, the results would have been different had we used a high-frequency intradaily returns as proposed by Andersen and Bollerslev (1997). Since, we do not have such data, we leave this issue for later discussion. Consequently, in the option pricing section, we only concentrate between the plain vanilla (Black-Scholes) and the jump-diffusion option pricing models.

5. Payoffs and Empirical Call Valuations

In this section, we start by outlining the payoffs and valuations for foreign equity index European call options that is struck in domestic currency. We then describe Merton's (1976a) Poisson Jump-Diffusion model for foreign index option. We present the errors in option pricing when jumps are ignored in the last section.

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the regression can explain upto 50% as compared to 6- 10 % which are reported previously.

5.1. Foreign Equity Index Call Struck in Domestic Currency with no Jumps: Model I

The products offered at the ÖTOB lead us to analyze the case where the investor wishes to receive any positive returns from the foreign market, but also wants to be certain that those returns are meaningful when translated back into his own currency. Consequently, when one wants to take into account the exchange rate as well as the equity dynamics in the option pricing, one must modify the standard Black-Scholes model.

The payoff for this scenario is

$$C_I = Max\left[S^* E^* - K, 0\right]$$

where K is now a domestic currency amount, representing translation of the foreign equity value into domestic terms.

Both the CTX and HTX indices are translated into \$U.S. dollars at the exchange rate prevailing at the time of exercise. Thus, the options are on a traded security denominated in dollars with a price SE that has the volatility of the security's price as $\sigma_{SE} = \sqrt{\sigma_S^2 + \sigma_E^2 + 2\rho_{SE}\sigma_E\sigma_S}$. Consequently, we can use the standard Black-Scholes model with σ replaced by σ_{SE} and S replaced by SE. The value of this European call, therefore, is

$$\text{price of } C_I = SE\mathcal{N}\left(d_1\right) - Ke^{-r(T-t)}\mathcal{N}\left(d_2\right) \tag{5.1}$$

$$
\begin{aligned}
\text{where} \quad & \mathcal{N}(\bullet) \text{ is the cumulative normal distribution} \\
d_1 &= \frac{Ln\,(S) + Ln\,(E) - Ln\,(K) + \left(r + \sigma_{SE}^2/2\right)(T-t)}{\sigma_{SE}\sqrt{T-t}} \\
d_2 &= d_1 - \sigma_{SE}\sqrt{T-t} \\
r &= \text{domestic risk-free rate} \\
\sigma_{SE} &= \sqrt{\sigma_S^2 + \sigma_E^2 + 2\rho_{SE}\sigma_E\sigma_S}
\end{aligned}
$$

Here, * denotes the value at expiration, T = expiration date, S = spot equity price in the foreign currency (e.g. Czech Kron) at time t, E = is the exchange rate ($U.S. per unit foreign currency), r = U.S. risk-free rate, K = strike price in the $U.S., and σ_S is the foreign equity price volatility, σ_E is the exchange rate volatility, and ρ_{SE} is the correlation between the foreign equity and exchange rate.

5.2. Poisson Jump Diffusion Model: Model II

Mapping equation (3.8b) with Merton's (1976a) option formula for a jump diffusion process, the European call option price under the product jump-diffusion model is

$$
\begin{aligned}
C_{II}^{J}\left(SE, T-t; r, \sigma^2, \lambda, \theta, \delta^2, K\right) &= \sum_{n=0}^{\infty} \frac{e^{-\lambda^*(T-t)}\left\{\lambda^*(T-t)\right\}^n}{n!} \\
&\bullet C_n \begin{pmatrix} SE, T-t;\ r - \lambda\left(e^{\theta+\left(\delta^2\right)/2} - 1\right) \\ +n\frac{\theta+\left(\delta^2/2\right)}{T-t},\ \sqrt{\widetilde{\sigma_{SE}^2} + \frac{n\delta^2}{T-t}},\ K \end{pmatrix}
\end{aligned}
\tag{5.2}
$$

where $\lambda^* = \lambda e^{\theta + (\delta^2/2)}$, $\lambda = (\lambda_S + \lambda_E)$, $\theta = (\theta_S + \theta_E)$, $\delta = (\delta_S^2 + \delta_E^2)$. C_n is the Black-Scholes price of a call with the volatility of $\sqrt{\widetilde{\sigma_{SE}^2} + \frac{n\delta^2}{T-t}}$ and interest rate of $r - \lambda \left(e^{\theta + (\delta^2)/2} - 1 \right) + n \frac{\theta + (\delta^2/2)}{T-t}$.

5.3. Implications for Option Pricing: Errors in Option Pricing

In our analysis, we focus on the option price differences between competing models and not on the discrepancies between the models and market option prices. In doing so, we sidestep the limitations of observing the actual prices and can apply the theory to establish what the prices of these claims *should* be. More specifically, we analyze the implications which arise for option pricing if an investor mistakenly neglects the jump-component in the underlying index returns. To measure the mispricings from such negligence, we use the relative difference in percentage, $\varepsilon = \frac{C_{II}^J(SE, T-t; r_f, \sigma^2, \lambda, \theta, \delta^2, K) - C_I^{BS}(SE, T-t; r, \sigma_{total}^2, K)}{C_{II}^J(SE, T-t; r, \sigma^2, \lambda, \theta, \delta^2, K)} \times 100$. C_{II}^J is calculated from equation (5.2) using the estimated values of $\widehat{\sigma}^2, \widehat{\lambda}, \widehat{\theta}$,and $\widehat{\delta}^2$ in Table 3[16]. And C_I^{BS} is computed from equation (5.1) with the total variance, $\sigma_{Total}^2 \equiv \widehat{\sigma}^2 + \widehat{\lambda} \left(\widehat{\theta}^2 + \widehat{\delta}^2 \right)$. The options are classified by time to maturity and the ratio of stock and strike prices, SE/K. We consider short term, medium term, and long term as 7 days (1 week), 30 days (4 weeks) and 90 days (12 weeks) respectively. The ratio SE/K is taken to represent in- (1.025), at- (1.00), and out (0.975) of the money classes. We also present option values for deep in- (1.05), and out (0.95) of the money classes. For the risk-free rate, we fixed $r = 0.02\%$ per daily.

[16] It should be pointed out that in computing C_{II}^J, we used $N = 200$ whereas $N = 10$ was used for estimating the Maximum Likelihood function. Since our estimated value for daily λ was quite large, using $N = 10$ would have degenerated the call value C_{II}^J to near zero.

We present both relative and absolute option pricing bias for the CTX and HTX using the parameters previously estimated. Tables 6 and 7 show the relative pricing errors, whereas Figures 2 and 3 show the absolute difference,

see Tables 6 and 7 and

Figures 2 and 3 in appendix.

A clear message from Tables 6 and 7 and Figures 3 and 4 is that at-the-money, the Black-Scholes call cannot underprice the jump model (Ball and Torous, 1985; Merton, 1976b). However, the Black-Scholes call value can underprice or even overprice in- and out-of-the-money call options, although the magnitude of some of these errors are minimal. Figures 3 and 4 show the well-known v-shape relationship in pricing bias with the mis-pricing decreasing as the time to maturity increases. For CTX and HTX, the Black-Scholes overprices short-term at-the-money. However, for short term in- and out-of-the money, the Black-Scholes either under- or overprices, depending on the skewness of the underlying security's return distribution. For example, HTX with positive skewness in its return distribution (0.23) and positive mean return of the jump component $\left(\widehat{\theta} = 0.103 \times 10^{-2}\right)$, the Black-Scholes underprices short term out-of-the-money but overprices in-the-money options. On the other hand, the Black-Scholes underprices in-the-money options but overprices out-of-the-money for CTX, which has a negative skewness with negative mean return of the jump component $\left(\widehat{\theta} = -0.0467 \times 10^{-2}\right)$.

In terms of the relative pricing bias, the Black-Scholes underprices short term out-of-the-money options approximately by 18 percent for HTX, but overprices about 4 percent for CTX. Further, underpricing of 12.8 and 6.01 percents by the Black-Scholes for HTX at 3 and 4 weeks deep out-of-the-money (0.95) are other noticeable figures. For HTX, a significant underpricing is also reported for the deep deep out-of-the-money (0.9) at 9 to 12 weeks[17]. These figures for HTX are the consequence of the large number of jumps and positive skewness in the underlying return distribution. As a result, the diffusion process (i.e. Black-Scholes model) underestimates the likelihood of jump (i.e. Merton's model) which would bring one of these short-term out-of-the-money call options into the money. The values for HTX are in line with the other research, namely, 12.8 percent underpricing reported by Jorion (1988) on the U.S. stock index options.[18]

Other than the out-of-the-money options, the pricing bias is quite small, which lends additional support for Ball and Torous (1985) analysis; there is a few operational discrepancies between Merton's Poisson jump and Black-Scholes pure diffusion models. The last remark on the operational aspect of the Jump model should be emphasized for the markets such as CTX and HTX. As mentioned previously, both CTX and HTX are thinly traded with high transaction costs. There is a fixed charge of 20 USD per ticket. The ÖTOB's clearing fee is 1.5 percent of the premium (Min.. 1 USD per contract) and other investment banks' fees are between 0.7 - 1 percent per contract with no minimum fee. For options exercised, the banks, including ÖTOB,

[17]These numbers for pricing errors must be interpreted with some caution. The boundary numbers in tables 5.1 and 5.2 (i.e.the numbers that borders zeros) are not robust. For example, if the cut off criteria is set as $(\frac{C_J}{K} - \frac{C_{BS}}{K}) < 0.005$, we would not obtain such numbers.

[18]Jorion (1988) also reported 17 percent underpricing on \$/DM currency options and Bodurtha and Courtadon (1987) reported 29 percent underpricing of the American option in the currency market.

charge 5 USD per contract.[19] With these transaction cost figures, except for the short term out-of-the-money options, one wonders whether the pricing discrepancies can actually be exploited.

6. Conclusion

In this paper, we analyze whether discontinuous and time varying volatility models can describe the data on the Czech Traded Index (CTX) and Hungarian Traded Index (HTX). From the casual and statistical evidence, the CTX and HTX do not display the log-normal distribution. The critical determinant of the price of a European stock option is the terminal stock price distribution. To use the Black-Scholes model, one must have the log-normal distribution. Consequently, Black-Scholes would tend to either under or over price in - or out-of-the money calls depending on the size of the tails of the distribution. We find that the Poisson Jump model statistically lend support to describe the data, and that the Poisson jump call option model can explain some of the empirically observed distributions and mispricings in the Czech and Hungarian Traded Indices options markets. Significant mispricings are observed for the short-term out-of-the-money call options; the Black-Scholes overprices about 4 percent for CTX and underprices about 18 percent for HTX. However, we question whether the mispricings from the jump model are operational, especially, in these emerging financial markets. These markets must improve in their liquidity and volume. Otherwise, some of the observed mispricings are less operative since thinness implies that few investors use options to hedge the risks associated with their respec-

[19] I thank Marcus Klug (at RZW Securities) for providing these figures.

tive cash investments.

There are still many unanswered and open questions for these markets. For example, we can further analyze the Jump model with the modification that we incorporate the time varying volatility in foreign equity index-linked options as in Bates (1996) and Bakshi, Cao and Chen (1998). Moreover, we could also explicitly incorporate the degree of transaction costs for these products. Nevertheless, our current results from the Jump option pricing model on CTX and HTX shed light on the basic properties of these markets, and lend support to the cause of further market development.

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Appendix

Figure 1: Time series of Czech (CTX) and Hungarian Indices (HTX) from 1.1.1995 to 1.7.1997.

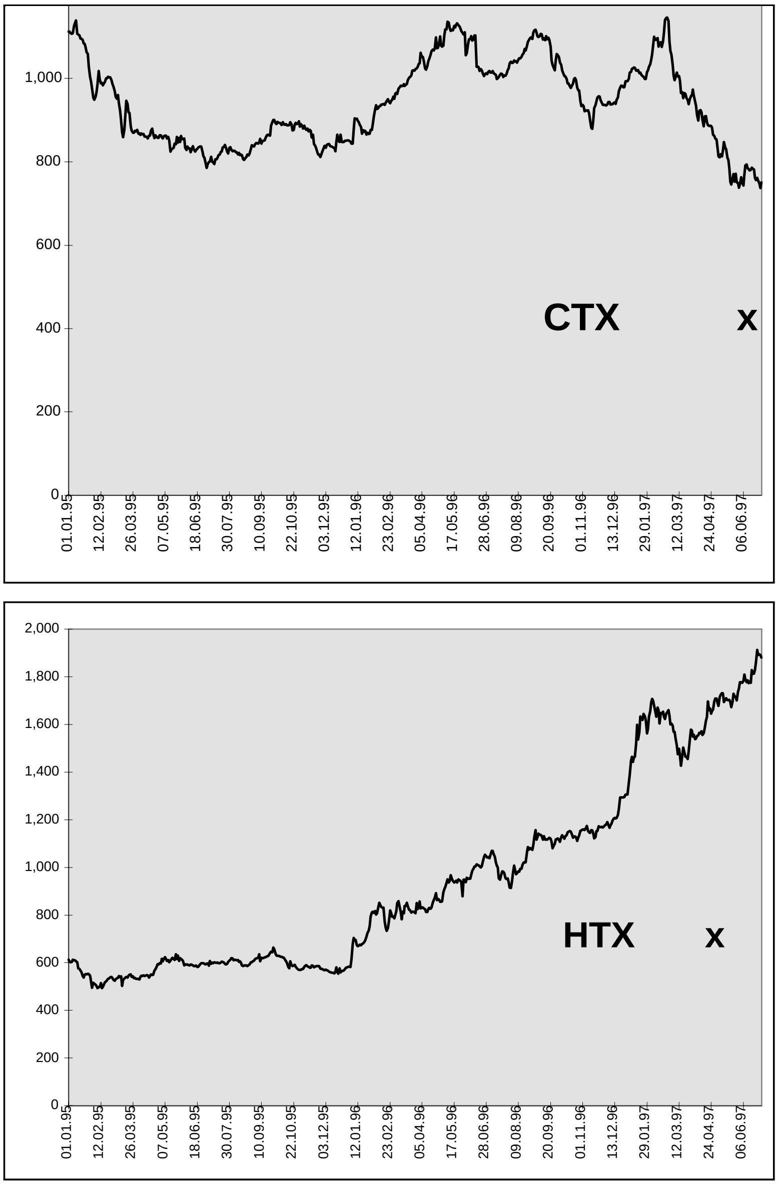

Figure 2: Absolute Pricing Error ($C^J/K - C^{BS}/K$) of Call Options on CTX.

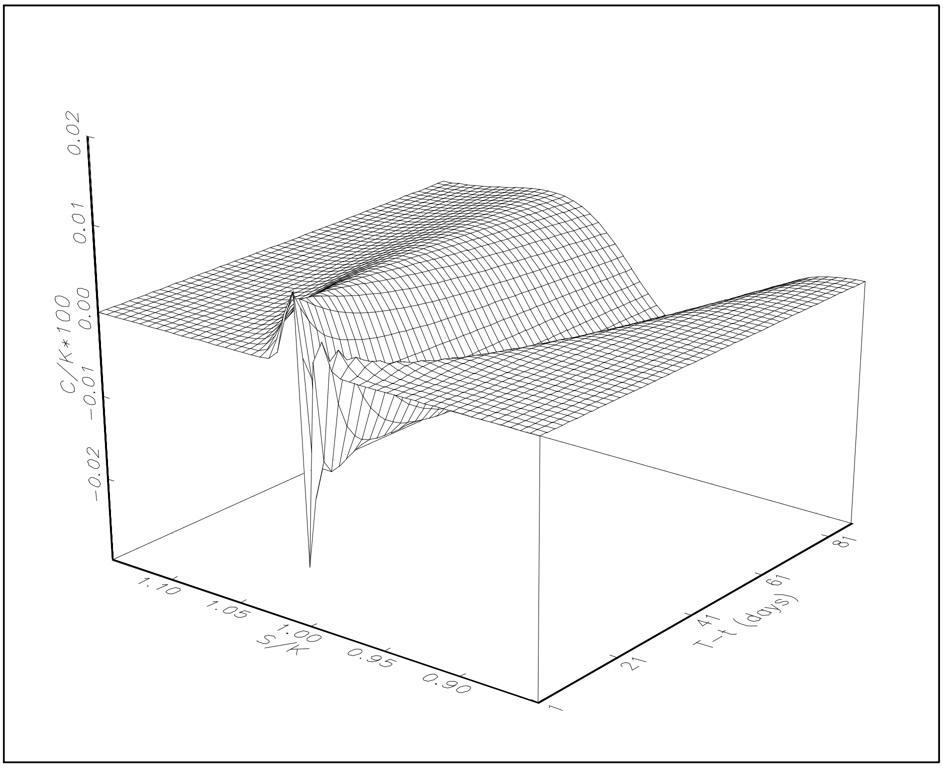

Figure 3: Absolute Pricing Error ($C^J/K - C^{BS}/K$) of Call Options on HTX.

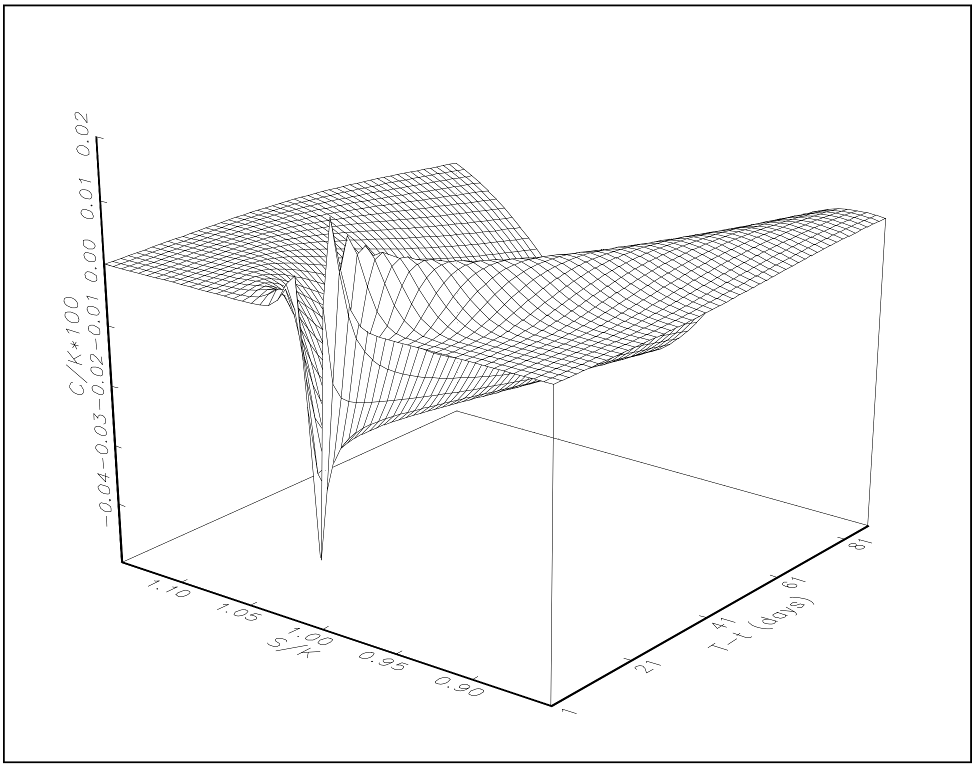

Table 1: Sample Statistics for Returns on Czech Indices (CTX, CTL, CZK) and on Hungarian Indices (HTX, HTL, HUF) from 4.1.1995 to 1.7.1997

	Czech			Hungary		
Statistics	CTX	CTL	CZK	HTX	HTL	HUF
Mean	-0.027%	-0.016%	0.010%	0.075%	0.111%	0.035%
Stdev.	0.503%	0.455%	0.297%	0.744%	0.675%	0.339%
Median	0.000%	0.000%	0.000%	0.024%	0.067%	0.019%
Maximum	1.681%	2.542%	3.602%	4.345%	4.146%	3.936%
Minimum	-3.092%	-3.233%	-1.200%	-3.288%	-3.288%	-1.438%
Skewness	-0.48	-0.58	2.78	0.23	0.41	2.30
z-value	-4.94	-6.01	28.8	2.39	4.28	23.8
p-value	0.00	0.00	0.00	0.01	0.00	0.00
Kurtosis	3.57	7.67	34.9	4.05	5.01	33.6
z-value	18.5	39.7	180.9	20.9	25.9	173.9
p-value	0.00	0.00	0.00	0.00	0.00	0.00

Table 2: Sample First Order Autocorrelation of Returns and of Squared Returns on Czech Indizes (CTX, CTL, CZK) and on Hungarian Indizes (HTX, HTL, HUF) from 4.1.1995 to 1.7.1997

	Czech			Hungary		
Statistics	CTX	CTL	CZK	HTX	HTL	HUF
$ACF_1(x)$	0.21	0.32	-0.15	0.054	0.21	-0.275
Q-value	29.10	69.31	14.55	1.90	29.62	49.08
p-value	0.00	0.00	0.00	0.16	0.00	0.00
$ACF_1(x^2)$	0.12	0.09	0.05	0.31	0.33	0.09
Q-value	10.59	5.17	1.93	60.60	69.73	5.37
p-value	0.00	0.02	0.16	0.00	0.00	0.02

Table 3: Maximum Likelihood Estimates for the parameters of the Plain Vanilla, the Stochastic Jump, the GARCH (1,1) and the combined Model for CTX and HTX from 4.1.1995 to 1.7.1997

	CTX				HTX			
	Vanilla	Jump	Garch	Comb	Vanilla	Jump	Garch	Comb
$\widehat{\mu}$	-0.0264	0.0044	0.0015	0.0216	0.0753	0.0079	0.0639	0.0194
	(0.0198)	(0.0190)	(0.0195)	(0.0161)	(0.0292)	(0.0274)	(0.0268)	(0.0240)
$\widehat{\sigma}^2$	0.2530	0.0554			0.5522	0.1079		
	(0.0141)	(0.0112)			(0.0307)	(0.0278)		
$\widehat{\theta}$		-0.0467		-0.1987		0.1031		0.1425
		(0.0371)		(0.1118)		(0.0550)		(0.1068)
$\widehat{\delta}^2$		0.2948		0.4816		0.6543		0.9041
		(0.0619)		(0.1724)		(0.1521)		(0.3239)
$\widehat{\lambda}$		0.6591		0.1683		0.6537		0.2402
		(0.1488)		(0.0698)		(0.1689)		(0.1030)
$\widehat{\alpha}$			0.0275	0.0189			0.3273	0.0847
			(0.0104)	(0.0060)			(0.0478)	(0.0209)
$\widehat{\eta}$			0.7695	0.2494			0.0444	0.4496
			(0.0.0632)	(0.0507)			(0.0888)	(0.0846)
$\widehat{\beta}$			0.1277	0.5292			0.3765	0.0814
			(0.0347)	(0.0738)			(0.0751)	(0.0694)

The parameters in this table are estimates for the respective models of percentual daily returns given by $x_t = 100 * \ln\left(\frac{y_t}{y_t - 1}\right)$ where y_t equals CTX_t and HTX_t respectivley. The values in brackets are the estimated standard errors of the point estimates.

Table 4: Comparison of Models: Log-Likelihood and Likelihood Ratio Statistics for the Test of Restricted against Unrestricted Models

	CTX				HTX			
	Vanilla	Jump	Garch	Comb	Vanilla	Jump	Garch	Comb
d	2	5	4	7	2	5	4	7
$L(X\|\gamma)$	-473.4	-421.2	-445.1	–389.3	-725.9	-661.7	-680.1	-621.1
Vanilla χ^2_{d-2}		104.4	56.5	167.9		128.2	91.6	209.5
Jump χ^2_{d-5}				63.5				81.2
Garch χ^2_{d-4}				111.4				117.9

The number of parameters (degrees of freedom) is given by $\mathcal{L}(X|\gamma)$ denotes the maximized log-likelihood function.The test statsistic χ^2 is Chi-Square distributed.

Table 5: Regression of Observed Variance on Predicted Variance according to the GARCH(1,1)-Model

	CTX	HTX
β_0	0.075	0.117
	(0.047)	(0.075)
β_1	0.689	0.775
	(0.158)	(0.098)
R^2	0.028	0.088

The underlying regression model is given by: $(x_t - \mu)^2 = \beta_0 + \beta_1 \widehat{\sigma}_t^2 + \eta_t$ where x_t are the observed returns, $\widehat{\sigma}_t^2$ is the predicted variance and η_t is white noise.

Table 6: Percentual Relative Pricing Error of Black Scholes Pricing for Call Options on CTX with respect to the Pricing according to the Jump Model.

	C/K								
	0.8	0.9	0.95	0.975	1	1.025	1.05	1.1	1.2
T-t (weeks)									
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0 .00
1	0.00	0.00	0.00	0.00	-2.37	0.15	0.00	0.00	0.00
2	0.00	0.00	0.00	0.00	-1.20	0.19	0.01	0.00	0.00
3	0.00	0.00	0.00	-4.02	-0.83	0.16	0.03	0.00	0.00
4	0.00	0.00	0.00	-3.43	-0.65	0.12	0.04	0.00	0.00
5	0.00	0.00	0.00	-2.82	-0.55	0.10	0.05	0.00	0.00
6	0.00	0.00	-4.04	-2.35	-0.47	0.07	0.05	0.00	0.00
7	0.00	0.00	-3.98	-2.00	-0.42	0.05	0.05	0.00	0.00
8	0.00	0.00	-3.72	-1.73	-0.38	0.04	0.05	0.00	0.00
9	0.00	0.00	-3.40	-1.52	-0.35	0.03	0.05	0.00	0.00
10	0.00	0.00	-3.10	-1.35	-0.33	0.02	0.05	0.00	0.00
11	0.00	0.00	-2.82	-1.22	-0.31	0.01	0.04	0.01	0.00
12	0.00	0.00	-2.58	-1.11	-0.29	0.00	0.04	0.01	0.00
13	0.00	0.00	-2.36	-1.02	-0.28	-0.01	0.04	0.01	0.00
14	0.00	0.00	-2.17	-0.94	-0.27	-0.01	0.03	0.01	0.00
15	0.00	0.00	-2.01	-0.87	-0.25	-0.02	0.03	0.01	0.00

The relative pricing error is given by: $\frac{C^J - C^{BS}}{C^J} \times 100$. If the price according to the Jump model is less than one promille of the strike price (i.e $C/K < 0.001$) the price according to the Jump model is set to be zero.

Table 7: Percentual Relative Pricing Error of Black Scholes Pricing for Call Options on HTX with respect to the Pricing according to the Jump Model.

					C/K				
	0.8	0.9	0.95	0.975	1	1.025	1.05	1.1	1.2
T-t (weeks)									
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0 .00
1	0.00	0.00	0.00	18.20	-3.32	-0.08	0.01	0.00	0.00
2	0.00	0.00	0.00	2.81	-2.00	-0.42	-0.01	0.00	0.00
3	0.00	0.00	12.80	0.12	-1.57	-0.55	-0.07	0.00	0.00
4	0.00	0.00	6.01	-0.66	-1.36	-0.59	-0.13	0.00	0.00
5	0.00	0.00	2.94	-0.96	-1.23	-0.60	-0.18	0.00	0.00
6	0.00	0.00	1.36	-1.08	-1.14	-0.61	-0.22	-0.01	0.00
7	0.00	0.00	0.46	-1.13	-1.07	-0.61	-0.25	-0.02	0.00
8	0.00	0.00	-0.08	-1.14	-1.02	-0.60	-0.27	-0.03	0.00
9	0.00	8.21	-0.42	-1.14	-0.98	-0.60	-0.28	-0.03	0.00
10	0.00	5.96	-0.64	-1.14	-0.95	-0.59	-0.30	-0.04	0.00
11	0.00	4.33	-0.80	-1.12	-0.92	-0.58	-0.31	-0.05	0.00
12	0.00	3.13	-0.90	-1.11	-0.89	-0.58	-0.32	-0.06	0.00
13	0.00	2.22	-0.97	-1.09	-0.87	-0.57	-0.32	-0.07	0.00
14	0.00	1.53	-1.02	-1.08	-0.85	-0.57	-0.33	-0.08	0.00
15	0.00	0.99	-1.06	-1.06	-0.83	-0.56	-0.33	-0.08	0.00

The relative pricing error is given by: $\frac{C^J - C^{BS}}{C^J} \times 100$. If the price according to the Jump model is less than one promille of the strike price (i.e $C/K < 0.001$) the price according to the Jump model is set to be zero.

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